# MATH 105 101 Midterm 1 Sample 4

1. (15 marks)

   (a) (4 marks) Given the function:

   $$f(x,y) = \arcsin x^2 + y,$$

   find its first order partial derivatives at the point $(0,3)$. *Simplify your answers.*

   (b) (3 marks) Find and sketch the domain of the function $f(x,y) = \frac{x}{\sqrt{4x^2+y^2-9}}$.*Label your intercepts.*

   (c) (3 marks) Let $\mathbf{v} = \langle 3,2,1\rangle$, and $\mathbf{w} = \langle 9,-6,2\rangle$. Are the two vectors $\mathbf{v}$ and $\mathbf{w}$ parallel, perpendicular, or neither? Justify your answer.

   (d) (2 marks) Can you find a plane parallel to the $xy$-plane (ie. the plane $z = 0$) passing through the point $P(1,2,-1)$? If yes, find the equation of this plane. If not, explain why not.

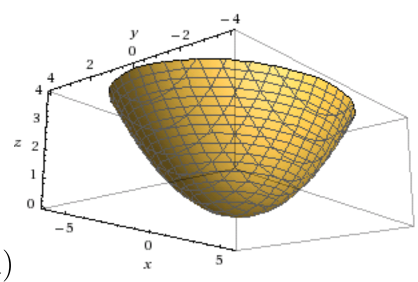
   (e) (3 marks) Is there a function $f(x,y)$ such that $\nabla f(x,y) = \langle \sin y - 1, x\cos y - x\rangle$? If not, explain why no such function exists; otherwise, find $f(x,y)$. Clearly state any result that you may use.

2. (5 marks) Consider the surface $S$ given by:

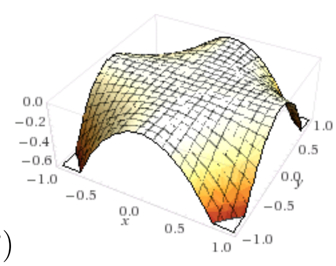
   $$z - \frac{x^2}{9} = \frac{y^2}{4}.$$

   (a) (4 marks) Find and sketch the traces of $S$ in the $y=0$ and $z=1$ planes.

   (b) (1 mark) Based on the traces you sketched above, which of the following renderings represents the graph of the surface?




(A)




(B)

3. (10 marks) Let $R$ be the semicircular region $\{x^2+y^2 \le 9, y \ge x\}$. Find the maximum and minimum values of the function

   $$f(x,y) = x^2 + y^2 - 4x.$$

   on the *boundary of the region* $R$.

4. (10 marks) Find *all* critical points of the following function:

$$f(x, y) = \frac{x^4}{4} + \frac{y^4}{4} - xy$$

lying in the region $\{(x, y) \mid y \geq 0\}$. Classify each point as a local minimum, local maximum, or saddle point. *You do not have to solve for extrema on the boundary.*

5. (10 marks) A paper company makes two kinds of paper, $x$ units of brown and $y$ units of white every month. Given a fixed amount of raw material, $x$ and $y$ must satisfy the production possibility curve:

$$4x^2 + 25y^2 = 50,000, \text{ for } x \geq 0, y \geq 0.$$

It costs the company \$23 to produce a single unit of brown paper, and \$42 to produce a unit of white. On the other hand, brown and white paper sell for \$25 per unit and \$52 per unit respectively. Assuming that the company is confident of selling all the units it produces, find how many units of brown and white paper it should manufacture every month so as to maximize its total profit, using the method of Lagrange multipliers.

Clearly state the objective function and the constraint. *You are not required to justify that the solution you obtained is the absolute maximum.* **A solution that does not use the method of Lagrange multipliers will receive no credit, even if it is correct.**